COUNTING TO CALCULUS

The Complete Topic List

From counting and place value to Calculus, Linear Algebra, Differential Equations, Data Science and the Mathematics of Machine Learning

Every topic in order, from the most basic to the most advanced. Each one is taught once.

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20	172	491	Grade 3 →
stages	units	numbered topics	and no upper limit

How to read this list

Numbered from basic to advanced. Topic 1 is counting to 100. Topic 470 is the final machine learning capstone project. Topics 471–491 form the open-ended "Beyond" stage.

Taught once. A topic appears once, at the point where it is first needed. Later topics build on it and do not re-teach it.

Grouped into units. The orange bar above each group names its unit, for example A5.8 Quadratic equations, and lists the units it needs first ("Needs").

Starting point. This list is written for a serious third grader who already knows addition, subtraction and the multiplication tables. Those topics appear as a quick recap.

The journey at a glance

Stage	Course	Band	Units	Topics
0	Counting Recap	Foundations	1	1–4
1	Whole Numbers & Place Value	Foundations	9	5–35
2	Fractions & Decimals	Foundations	7	36–51
3	Measurement, Geometry & Data Foundations	Foundations	7	52–60
4	Pre-Algebra	Foundations	15	61–97
5	Algebra 1	High School Core	14	98–134
6	Geometry	High School Core	12	135–161
7	Algebra 2	High School Core	10	162–188
8	Trigonometry	High School Core	8	189–207
9	Precalculus	High School Core	7	208–223
10	Calculus 1	University Core	8	224–248
11	Calculus 2	University Core	7	249–268
12	Calculus 3	University Core	8	269–292
13	Linear Algebra	University Core	8	293–320
14	Discrete Mathematics	University Core	8	321–346

Stage	Course	Band	Units	Topics
15	Differential Equations	University Core	8	347–366
16	Probability & Statistics (calculus-based)	University Core	10	367–397
17	Data Science (OpenStax Principles of Data Science)	AI Capstone	10	398–429
18	Capstone: Mathematics of Machine Learning	AI Capstone	10	430–470
19	Beyond: No Limits (optional)	Beyond	5	471–491

STAGE 0 · FOUNDATIONS

Counting Recap

Topics 1–4 · 1 unit

History: Tally marks and the first counting tools of ancient history. **Where it is used:** Counting money, objects, time.

N0.1 Counting 1-100 recap *Needs: Start here*	
1	Count to 100 forward and back
2	Skip count by 2, 5, 10
3	Number line
4	Even and odd

STAGE 1 · FOUNDATIONS

Whole Numbers & Place Value

Topics 5–35 · 9 units

History: Hindu-Arabic numerals: Brahmagupta's zero, al-Khwarizmi, Fibonacci's Liber Abaci. **Where it is used:** Reading large numbers in budgets, populations, distances.

N1.1 Place value to 1,000	*Needs: N0.1 Counting 1-100 recap*
5	Hundreds, tens, ones
6	Expanded form
7	Compare and order
8	Number line
N1.2 Place value to billions; rounding	*Needs: N1.1 Place value to 1,000*
9	Periods
10	Standard, word and expanded form
11	Rounding
12	Roman numerals
N1.3 Addition & subtraction recap	*Needs: N1.2 Place value to billions; rounding*
13	Standard vertical algorithm with regrouping
14	Estimation check (recap only, already mastered)
N1.4 Multiplication facts recap & properties	*Needs: N1.3 Addition & subtraction recap*
15	Tables 0-12 fluency check
16	Commutative, associative, distributive properties
N1.5 Multi-digit multiplication	*Needs: N1.4 Multiplication facts recap & properties*
17	Standard vertical algorithm
18	Multiplying by 10s and 100s
19	Estimation
N1.6 Division & long division	*Needs: N1.5 Multi-digit multiplication*
20	Division facts
21	Long division (standard algorithm)
22	Remainders
23	Checking by multiplication
N1.7 Order of operations	*Needs: N1.6 Division & long division*
24	PEMDAS
25	Grouping symbols
26	Number sentences
N1.8 Factors, multiples & primes	*Needs: N1.7 Order of operations*
27	Divisibility rules
28	Prime and composite

29	Sieve of Eratosthenes
30	Prime factorisation
31	GCF
32	LCM
N1.9 Whole-number exponents	*Needs: N1.8 Factors, multiples & primes*
33	Powers
34	Square numbers
35	Powers of ten

STAGE 2 · FOUNDATIONS

Fractions & Decimals

Topics 36–51 · 7 units

History: Egyptian unit fractions (Rhind papyrus); Simon Stevin's decimals. **Where it is used:** Recipes, measurement, money, sharing.

F2.1 Meaning of fractions	*Needs: N1.9 Whole-number exponents · N1.8 Factors, multiples & primes*
36	Fractions on the number line
37	Equivalent fractions
38	Simplest form
F2.2 Comparing fractions; mixed numbers	*Needs: F2.1 Meaning of fractions*
39	Compare and order
40	Improper fractions and mixed numbers
F2.3 Adding & subtracting fractions	*Needs: F2.2 Comparing fractions; mixed numbers · N1.8 Factors, multiples & primes*
41	Least common denominator
42	Mixed numbers
43	Borrowing
F2.4 Multiplying & dividing fractions	*Needs: F2.3 Adding & subtracting fractions*
44	Fraction of a number
45	Reciprocals
46	Mixed numbers
F2.5 Decimal place value	*Needs: F2.4 Multiplying & dividing fractions · N1.2 Place value to billions; rounding*
47	Tenths to thousandths
48	Compare, round
49	Fraction-decimal conversion
F2.6 Decimal operations	*Needs: F2.5 Decimal place value · N1.6 Division & long division*
50	Add, subtract, multiply, long division with decimals
F2.7 Fraction & decimal problem solving	*Needs: F2.6 Decimal operations*
51	Multi-step word problems

STAGE 3 · FOUNDATIONS

Measurement, Geometry & Data Foundations

Topics 52–60 · 7 units

History: Eratosthenes measures the Earth; Archimedes and pi. **Where it is used:** Building, maps, sports statistics.

M3.1 Measurement & conversions	*Needs: F2.7 Fraction & decimal problem solving · F2.6 Decimal operations*
52	Time, money, US customary and metric units, conversions
M3.2 Perimeter & area	*Needs: M3.1 Measurement & conversions*
53	Rectangles, triangles, parallelograms, trapezoids, composite figures
M3.3 Lines, angles & shapes	*Needs: M3.2 Perimeter & area*
54	Measuring angles with a protractor
55	Classifying triangles, quadrilaterals, polygons
M3.4 Circles	*Needs: M3.3 Lines, angles & shapes*
56	Radius, diameter, circumference, area, pi
M3.5 Volume & surface area	*Needs: M3.4 Circles*
57	Prisms, cylinders, nets
M3.6 Coordinate plane	*Needs: M3.5 Volume & surface area*
58	Ordered pairs
59	Four quadrants
M3.7 Data displays	*Needs: M3.6 Coordinate plane*
60	Tally charts, bar graphs, line plots, line graphs, pictographs

STAGE 4 · FOUNDATIONS

Pre-Algebra

Topics 61–97 · 15 units

History: Pythagoras; Diophantus' Arithmetica; Brahmagupta's rules for negative numbers. **Where it is used:** Discounts, interest, scale models, temperature.

P4.1 Ratios, rates & proportions	*Needs: M3.7 Data displays · F2.4 Multiplying & dividing fractions*
61	Unit rates
62	Proportions
63	Cross-multiplication
P4.2 Percent	*Needs: P4.1 Ratios, rates & proportions · F2.6 Decimal operations*
64	Percent of
65	Increase and decrease
66	Discount, tax, simple interest
P4.3 Integers	*Needs: P4.2 Percent*
67	Number line
68	Absolute value
69	Four operations with integers
P4.4 Rational numbers	*Needs: P4.3 Integers*
70	Signed fractions and decimals
71	Repeating decimals
P4.5 Exponent rules & scientific notation	*Needs: P4.4 Rational numbers · N1.9 Whole-number exponents*
72	Integer exponents
73	Laws of exponents
74	Scientific notation
P4.6 Roots & the real numbers	*Needs: P4.5 Exponent rules & scientific notation*
75	Square and cube roots
76	Irrational numbers
77	The real number system
P4.7 Variables & expressions	*Needs: P4.6 Roots & the real numbers · N1.7 Order of operations*
78	Evaluating
79	Distributive property
80	Combining like terms
P4.8 One- & two-step equations and inequalities	*Needs: P4.7 Variables & expressions*
81	Undo in reverse order (SADMEP)
82	Graphing inequalities
P4.9 Multi-step equations	*Needs: P4.8 One- & two-step equations and inequalities*
83	Variables on both sides

84	Literal equations
P4.10 Pythagorean theorem	*Needs: P4.9 Multi-step equations · P4.6 Roots & the real numbers · M3.6 Coordinate plane*
85	Theorem and converse
86	Distance between points
P4.11 Transformations & similarity	*Needs: P4.10 Pythagorean theorem · M3.6 Coordinate plane*
87	Translations, reflections, rotations, dilations
88	Scale drawings
P4.12 Angle relationships	*Needs: P4.11 Transformations & similarity · M3.3 Lines, angles & shapes*
89	Parallel lines and transversals
90	Triangle angle sum
P4.13 Cones, spheres & pyramids	*Needs: P4.12 Angle relationships · M3.5 Volume & surface area*
91	Volume formulas
P4.14 Introductory probability	*Needs: P4.13 Cones, spheres & pyramids · F2.4 Multiplying & dividing fractions*
92	Simple and compound events
93	Sample spaces
P4.15 Introductory statistics	*Needs: P4.14 Introductory probability · M3.7 Data displays*
94	Mean, median, mode, range
95	Box plots
96	Histograms
97	Sampling

STAGE 5 · HIGH SCHOOL CORE

Algebra 1

Topics 98–134 · 14 units

History: al-Khwarizmi's al-jabr; Descartes joins algebra and geometry. **Where it is used:** Phone plans, break-even points, projectile paths, population growth.

A5.1 Functions	*Needs: P4.15 Introductory statistics · P4.9 Multi-step equations · M3.6 Coordinate plane*
98	Relations
99	Function notation
100	Domain and range
A5.2 Linear functions	*Needs: A5.1 Functions*
101	Slope
102	Intercepts
103	Slope-intercept, point-slope, standard form
104	Graphing
A5.3 Linear modeling	*Needs: A5.2 Linear functions · P4.15 Introductory statistics*
105	Parallel and perpendicular lines
106	Scatter plots
107	Line of best fit
108	Correlation
A5.4 Systems of equations	*Needs: A5.3 Linear modeling*
109	Graphing, substitution, elimination
110	Systems of inequalities
A5.5 Inequalities & absolute value	*Needs: A5.4 Systems of equations · P4.8 One- & two-step equations and inequalities*
111	Compound inequalities
112	Absolute value equations and inequalities
A5.6 Polynomials	*Needs: A5.5 Inequalities & absolute value · P4.5 Exponent rules & scientific notation*
113	Add, subtract, multiply
114	Special products
A5.7 Factoring	*Needs: A5.6 Polynomials*
115	GCF
116	Trinomials
117	Difference of squares
118	Grouping
A5.8 Quadratic equations	*Needs: A5.7 Factoring · A5.11 Radicals*
119	Factoring
120	Square roots
121	Completing the square

122	Quadratic formula
123	Discriminant
A5.9 Quadratic functions	*Needs: A5.8 Quadratic equations*
124	Graphs
125	Vertex form
126	Transformations
A5.10 Exponential functions	*Needs: A5.9 Quadratic functions · P4.2 Percent*
127	Growth and decay
128	Compound interest
A5.11 Radicals	*Needs: A5.10 Exponential functions · P4.6 Roots & the real numbers*
129	Simplifying
130	Operations
131	Radical equations
A5.12 Rational expressions	*Needs: A5.11 Radicals · A5.7 Factoring*
132	Simplify, multiply, divide
A5.13 Arithmetic & geometric sequences	*Needs: A5.12 Rational expressions*
133	Explicit and recursive formulas
A5.14 Piecewise & step functions	*Needs: A5.13 Arithmetic & geometric sequences*
134	Piecewise, step and absolute value functions

STAGE 6 · HIGH SCHOOL CORE

Geometry

Topics 135–161 · 12 units

History: Euclid's Elements: 2,000 years of proof. **Where it is used:** Architecture, engineering drawings, GPS.

G6.1 Foundations & constructions	*Needs: A5.14 Piecewise & step functions · P4.12 Angle relationships*
135	Points, lines, planes
136	Compass-and-straightedge constructions
G6.2 Logic & proof	*Needs: G6.1 Foundations & constructions*
137	Conditionals
138	Inductive and deductive reasoning
139	Two-column and paragraph proofs
G6.3 Parallel & perpendicular proofs	*Needs: G6.2 Logic & proof · A5.3 Linear modeling*
140	Proving lines parallel
141	Slope criteria
G6.4 Triangle congruence	*Needs: G6.3 Parallel & perpendicular proofs*
142	SSS, SAS, ASA, AAS, HL
143	CPCTC
G6.5 Triangle relationships	*Needs: G6.4 Triangle congruence*
144	Bisectors, medians, altitudes
145	Centers
146	Triangle inequalities
G6.6 Similarity	*Needs: G6.5 Triangle relationships · P4.11 Transformations & similarity*
147	AA, SSS, SAS similarity
148	Proportional parts
G6.7 Right triangles	*Needs: G6.6 Similarity · P4.10 Pythagorean theorem*
149	Special right triangles
150	Geometric mean
G6.8 Quadrilaterals & polygons	*Needs: G6.7 Right triangles*
151	Properties and proofs
152	Angle sums
G6.9 Circles	*Needs: G6.8 Quadrilaterals & polygons*
153	Arcs, chords, tangents, inscribed angles
154	Equation of a circle
G6.10 Area & volume derived	*Needs: G6.9 Circles · P4.13 Cones, spheres & pyramids*
155	Cavalieri's principle
156	Surface area
157	Similar solids

G6.11 Coordinate geometry	*Needs: G6.10 Area & volume derived*
158	Coordinate proofs
159	Transformations as functions
160	Symmetry
G6.12 Geometric probability	*Needs: G6.11 Coordinate geometry · P4.14 Introductory probability*
161	Length and area models

STAGE 7 · HIGH SCHOOL CORE

Algebra 2

Topics 162–188 · 10 units

History: Cardano and the cubic; Napier's logarithms; Euler's e. **Where it is used:** Earthquake scales, pH, compound growth, signal processing.

B7.1 Complex numbers	*Needs: G6.12 Geometric probability · A5.8 Quadratic equations*
162	The imaginary unit i
163	Operations with complex numbers
164	Complex roots of quadratics
B7.2 Polynomial functions	*Needs: B7.1 Complex numbers · A5.7 Factoring*
165	Long and synthetic division
166	Remainder and factor theorems
167	Rational roots
168	Fundamental Theorem of Algebra
169	End behavior
B7.3 Rational functions	*Needs: B7.2 Polynomial functions · A5.12 Rational expressions*
170	Asymptotes
171	Graphs
172	Rational equations
B7.4 Inverse & radical functions	*Needs: B7.3 Rational functions · A5.11 Radicals*
173	Composition
174	Inverses
175	Rational exponents
B7.5 Exponentials & logarithms	*Needs: B7.4 Inverse & radical functions · A5.10 Exponential functions · P4.5 Exponent rules & scientific notation*
176	Log laws
177	e and natural log
178	Exponential and log equations
B7.6 Systems in three variables	*Needs: B7.5 Exponentials & logarithms · A5.4 Systems of equations*
179	Elimination in 3 variables
B7.7 Function transformations	*Needs: B7.6 Systems in three variables*
180	Shifts, stretches, reflections for every family
B7.8 Series	*Needs: B7.7 Function transformations · A5.13 Arithmetic & geometric sequences*
181	Sigma notation
182	Arithmetic and geometric series
183	Infinite geometric series
B7.9 Counting & the binomial theorem	*Needs: B7.8 Series · P4.14 Introductory probability*
184	Permutations

185	Combinations
186	Pascal's triangle
187	Binomial theorem

B7.10 Nonlinear systems *Needs: B7.9 Counting & the binomial theorem*

188	Line-curve and curve-curve systems

STAGE 8 · HIGH SCHOOL CORE

Trigonometry

Topics 189–207 · 8 units

History: Hipparchus, Ptolemy, Aryabhata, Regiomontanus. **Where it is used:** Surveying, navigation, sound and light waves.

T8.1 Right-triangle trigonometry	*Needs: B7.10 Nonlinear systems · G6.7 Right triangles · G6.6 Similarity*
189	SOH-CAH-TOA
190	Angles of elevation and depression
T8.2 Radian measure	*Needs: T8.1 Right-triangle trigonometry · M3.4 Circles*
191	Radians
192	Arc length
193	Sector area
194	Angular speed
T8.3 The unit circle	*Needs: T8.2 Radian measure*
195	Trig functions of any angle
196	Reference angles
T8.4 Graphs of trig functions	*Needs: T8.3 The unit circle · B7.7 Function transformations*
197	Amplitude, period, phase shift
198	Reciprocal functions
199	Periodic modeling
T8.5 Inverse trig functions	*Needs: T8.4 Graphs of trig functions · B7.4 Inverse & radical functions*
200	Restricted domains
201	Evaluation
T8.6 Trig identities	*Needs: T8.5 Inverse trig functions*
202	Pythagorean, sum and difference, double and half angle
203	Proving identities
T8.7 Trig equations	*Needs: T8.6 Trig identities*
204	Solving on intervals and in general
T8.8 Oblique triangles	*Needs: T8.7 Trig equations*
205	Law of Sines
206	Law of Cosines
207	Area

STAGE 9 · HIGH SCHOOL CORE

Precalculus

Topics 208–223 · 7 units

History: Apollonius' conics; Fermat and Descartes. **Where it is used:** Satellite orbits, robotics, computer graphics.

C9.1 Function analysis	*Needs: T8.8 Oblique triangles · B7.7 Function transformations*
208	Even and odd
209	Increasing and decreasing
210	Average rate of change
C9.2 Conic sections	*Needs: C9.1 Function analysis · G6.9 Circles · A5.9 Quadratic functions*
211	Parabolas, ellipses, hyperbolas
C9.3 Parametric equations	*Needs: C9.2 Conic sections · T8.4 Graphs of trig functions*
212	Parametrizing curves
213	Eliminating the parameter
C9.4 Polar coordinates & polar complex numbers	*Needs: C9.3 Parametric equations · B7.1 Complex numbers · T8.3 The unit circle*
214	Polar graphs
215	Trigonometric form
216	De Moivre's theorem
C9.5 Vectors in the plane	*Needs: C9.4 Polar coordinates & polar complex numbers · T8.8 Oblique triangles*
217	Operations
218	Dot product
219	Applications
C9.6 Induction & convergence	*Needs: C9.5 Vectors in the plane · B7.8 Series*
220	Mathematical induction
221	Convergence intuition
C9.7 Introduction to limits	*Needs: C9.6 Induction & convergence*
222	Numerical and graphical limits
223	Continuity

STAGE 10 · UNIVERSITY CORE

Calculus 1

Topics 224–248 · 8 units

History: Newton and Leibniz; Cauchy makes limits rigorous. **Where it is used:** Velocity, optimization in business and engineering, rates of change.

K11.1 Limits & continuity	*Needs: C9.7 Introduction to limits*
224	Limit laws
225	One-sided and infinite limits
226	Epsilon-delta introduction
227	Intermediate Value Theorem
K11.2 The derivative	*Needs: K11.1 Limits & continuity*
228	Definition
229	Power, product, quotient and chain rules
K11.3 Transcendental derivatives	*Needs: K11.2 The derivative · B7.5 Exponentials & logarithms · T8.5 Inverse trig functions*
230	Trig, exponential, log and inverse functions
231	Implicit differentiation
K11.4 Related rates	*Needs: K11.3 Transcendental derivatives*
232	Modeling changing quantities
K11.5 Applications of derivatives	*Needs: K11.4 Related rates*
233	Extrema
234	Mean Value Theorem
235	Curve sketching
236	Optimization
237	L'Hopital
238	Newton's method
239	Linearization
K11.6 The integral	*Needs: K11.5 Applications of derivatives · B7.8 Series*
240	Antiderivatives
241	Riemann sums
242	Definite integral
243	Fundamental Theorem of Calculus
K11.7 Substitution	*Needs: K11.6 The integral*
244	u-substitution
245	Exponential and log integrals
K11.8 Applications of integration	*Needs: K11.7 Substitution · G6.10 Area & volume derived*
246	Area between curves
247	Volumes by disk, washer, shell

248	Average value

STAGE 11 · UNIVERSITY CORE

Calculus 2

Topics 249–268 · 7 units

History: The Bernoullis, Brook Taylor and Euler's series. **Where it is used:** Physics of work and fluids, approximation in computing.

K12.1 Physical applications	*Needs: K11.8 Applications of integration*
249	Arc length
250	Surface area
251	Work
252	Center of mass
253	Fluid force
K12.2 Techniques of integration	*Needs: K12.1 Physical applications · T8.6 Trig identities*
254	Integration by parts
255	Trig integrals
256	Trig substitution
257	Partial fractions
K12.3 Numerical & improper integrals	*Needs: K12.2 Techniques of integration*
258	Trapezoid and Simpson's rules
259	Improper integrals
K12.4 First differential equations	*Needs: K12.3 Numerical & improper integrals*
260	Slope fields
261	Separable equations
262	Euler's method
263	Logistic growth
K12.5 Sequences & series	*Needs: K12.4 First differential equations · C9.6 Induction & convergence*
264	Convergence tests: integral, comparison, ratio, root, alternating
K12.6 Power & Taylor series	*Needs: K12.5 Sequences & series*
265	Radius of convergence
266	Taylor and Maclaurin series
267	Error bounds
K12.7 Parametric & polar calculus	*Needs: K12.6 Power & Taylor series · C9.3 Parametric equations · C9.4 Polar coordinates & polar complex numbers*
268	Derivatives, arc length and area in parametric and polar form

STAGE 12 · UNIVERSITY CORE

Calculus 3

Topics 269–292 · 8 units

History: Lagrange, Gauss, Green and Stokes. **Where it is used:** Fluid flow, electromagnetism, 3D graphics, machine-learning gradients.

K13.1	**Vectors & geometry of space**	*Needs: K12.7 Parametric & polar calculus · C9.5 Vectors in the plane*
269	3D coordinates	
270	Dot and cross products	
271	Lines and planes	
K13.2	**Vector-valued functions**	*Needs: K13.1 Vectors & geometry of space*
272	Curves	
273	Curvature	
274	Motion in space	
K13.3	**Partial derivatives**	*Needs: K13.2 Vector-valued functions*
275	Functions of several variables	
276	Limits	
277	Partial derivatives	
K13.4	**Gradient & chain rule**	*Needs: K13.3 Partial derivatives*
278	Tangent planes	
279	Multivariable chain rule	
280	Directional derivatives	
281	Gradient	
K13.5	**Multivariable optimization**	*Needs: K13.4 Gradient & chain rule*
282	Critical points	
283	Second-derivative test	
284	Lagrange multipliers	
K13.6	**Multiple integrals**	*Needs: K13.5 Multivariable optimization*
285	Double and triple integrals	
286	Polar, cylindrical, spherical	
287	Jacobian	
K13.7	**Vector fields & line integrals**	*Needs: K13.6 Multiple integrals*
288	Conservative fields	
289	Green's theorem	
K13.8	**Surface integrals**	*Needs: K13.7 Vector fields & line integrals*
290	Flux	
291	Stokes' theorem	
292	Divergence theorem	

STAGE 13 · UNIVERSITY CORE

Linear Algebra

Topics 293–320 · 8 units

History: Gauss's elimination; Cayley and Sylvester's matrices. **Where it is used:** Computer graphics, search engines, every neural network.

L14.1	**Linear systems**	*Needs: K13.8 Surface integrals · B7.6 Systems in three variables*
293	Gaussian and Gauss-Jordan elimination	
294	Row echelon form	
L14.2	**Matrix algebra**	*Needs: L14.1 Linear systems*
295	Products	
296	Inverses	
297	Elementary matrices	
298	LU factorization	
L14.3	**Determinants**	*Needs: L14.2 Matrix algebra*
299	Cofactor expansion	
300	Properties	
301	Cramer's rule	
L14.4	**Vector spaces**	*Needs: L14.3 Determinants · K13.1 Vectors & geometry of space*
302	Subspaces	
303	Span	
304	Linear independence	
305	Basis	
306	Dimension	
307	Rank	
L14.5	**Linear transformations**	*Needs: L14.4 Vector spaces*
308	Kernel and range	
309	Matrix of a transformation	
310	Change of basis	
L14.6	**Eigenvalues & eigenvectors**	*Needs: L14.5 Linear transformations*
311	Characteristic polynomial	
312	Diagonalization	
L14.7	**Orthogonality & least squares**	*Needs: L14.6 Eigenvalues & eigenvectors*
313	Projections	
314	Gram-Schmidt	
315	QR	
316	Least squares	
L14.8	**Symmetric matrices & SVD**	*Needs: L14.7 Orthogonality & least squares*

317	Spectral theorem
318	Quadratic forms
319	Singular value decomposition
320	PCA

STAGE 14 · UNIVERSITY CORE

Discrete Mathematics

Topics 321–346 · 8 units

History: Boole's logic; Euler's bridges of Konigsberg; Turing. **Where it is used:** Computer science, cryptography, networks, algorithms.

D15.1 Logic	*Needs: L14.8 Symmetric matrices & SVD · G6.2 Logic & proof*
321	Propositional and predicate logic
322	Quantifiers
D15.2 Proof techniques	*Needs: D15.1 Logic · C9.6 Induction & convergence*
323	Direct, contrapositive, contradiction
324	Strong and structural induction
D15.3 Sets, functions & relations	*Needs: D15.2 Proof techniques*
325	Set operations
326	Equivalence relations
327	Partial orders
328	Cardinality
D15.4 Advanced counting	*Needs: D15.3 Sets, functions & relations · B7.9 Counting & the binomial theorem*
329	Pigeonhole
330	Inclusion-exclusion
331	Recurrence relations
332	Generating functions
D15.5 Number theory & cryptography	*Needs: D15.4 Advanced counting · N1.8 Factors, multiples & primes*
333	Modular arithmetic
334	Euclidean algorithm
335	Congruences
336	RSA
D15.6 Graph theory	*Needs: D15.5 Number theory & cryptography*
337	Euler and Hamilton paths
338	Trees
339	Shortest paths
340	Coloring
D15.7 Algorithms & complexity	*Needs: D15.6 Graph theory*
341	Big-O
342	Searching and sorting
343	Recursion
D15.8 Boolean algebra & automata	*Needs: D15.7 Algorithms & complexity*
344	Logic circuits

345	Finite automata
346	Formal languages

STAGE 15 · UNIVERSITY CORE

Differential Equations

Topics 347–366 · 8 units

History: Euler, Laplace and Fourier. **Where it is used:** Circuits, vibrations, epidemics, heat flow, control systems.

E16.1 First-order equations	*Needs: D15.8 Boolean algebra & automata · K12.4 First differential equations*
347	Linear (integrating factor), exact, Bernoulli
348	Existence and uniqueness
349	Modeling
E16.2 Second-order linear equations	*Needs: E16.1 First-order equations*
350	Characteristic equation
351	Undetermined coefficients
352	Variation of parameters
E16.3 Oscillations	*Needs: E16.2 Second-order linear equations*
353	Mechanical vibrations
354	RLC circuits
355	Resonance
E16.4 Laplace transforms	*Needs: E16.3 Oscillations*
356	Transforms
357	Step and impulse functions
358	Convolution
E16.5 Systems & stability	*Needs: E16.4 Laplace transforms · L14.6 Eigenvalues & eigenvectors*
359	Eigenvalue method
360	Phase planes
361	Stability
E16.6 Series solutions	*Needs: E16.5 Systems & stability · K12.6 Power & Taylor series*
362	Power-series solutions near ordinary points
E16.7 Numerical methods	*Needs: E16.6 Series solutions*
363	Improved Euler
364	Runge-Kutta
E16.8 Fourier series & PDEs	*Needs: E16.7 Numerical methods*
365	Fourier series
366	Heat and wave equations

STAGE 16 · UNIVERSITY CORE

Probability & Statistics (calculus-based)

Topics 367–397 · 10 units

History: Pascal and Fermat; Bayes; Gauss; Fisher. **Where it is used:** Medicine, polling, quality control, the foundation of machine learning.

R17.1 Study design	*Needs: E16.8 Fourier series & PDEs · P4.15 Introductory statistics*
367	Sampling methods
368	Experiments vs observational studies
369	Bias
R17.2 Exploring data	*Needs: R17.1 Study design*
370	Distributions
371	Standard deviation
372	z-scores
373	Outliers
R17.3 Probability	*Needs: R17.2 Exploring data · B7.9 Counting & the binomial theorem*
374	Axioms
375	Conditional probability
376	Independence
377	Bayes' theorem
R17.4 Discrete random variables	*Needs: R17.3 Probability*
378	Expectation
379	Variance
380	Binomial, geometric, Poisson
R17.5 Continuous & joint distributions	*Needs: R17.4 Discrete random variables · K13.6 Multiple integrals*
381	PDFs and CDFs by integration
382	Uniform, exponential, normal
383	Covariance
R17.6 Limit theorems	*Needs: R17.5 Continuous & joint distributions*
384	Sampling distributions
385	Law of Large Numbers
386	Central Limit Theorem
R17.7 Estimation	*Needs: R17.6 Limit theorems*
387	Estimators
388	Maximum likelihood
389	Confidence intervals
R17.8 Hypothesis testing	*Needs: R17.7 Estimation*
390	z, t and chi-square tests

391	p-values
392	Errors
393	Power
R17.9 Regression	*Needs: R17.8 Hypothesis testing · L14.7 Orthogonality & least squares*
394	Least squares
395	Inference for regression
396	Multiple regression in matrix form
R17.10 Bayesian inference	*Needs: R17.9 Regression*
397	Priors, likelihoods, posteriors

STAGE 17 · AI CAPSTONE

Data Science (OpenStax Principles of Data Science)

Topics 398–429 · 10 units

History: Florence Nightingale's charts; John Tukey's exploratory data analysis. **Where it is used:** Public health dashboards, business forecasting, sports analytics.

DS1 Ch.1 What are data & data science; Python	*Needs: R17.10 Bayesian inference · D15.7 Algorithms & complexity*
398	Data science cycle
399	Data types
400	Datasets
401	Python, Jupyter, NumPy
DS2 Ch.2 Collecting & preparing data	*Needs: DS1 Ch.1 What are data & data science; Python*
402	Collection
403	Cleaning
404	Missing data
405	Pandas
DS3 Ch.3 Descriptive statistics (Python only)	*Needs: DS2 Ch.2 Collecting & preparing data · R17.5 Continuous & joint distributions*
406	Descriptive statistics in Python (theory mastered in Stage 16)
407	Probability distributions in Python
DS4 Ch.4 Inference & regression (Python only)	*Needs: DS3 Ch.3 Descriptive statistics (Python only) · R17.9 Regression*
408	Confidence intervals and hypothesis tests in Python (theory mastered in Stage 16)
409	Regression analysis in Python
DS5 Ch.5 Time series & forecasting	*Needs: DS4 Ch.4 Inference & regression (Python only)*
410	Trends
411	Seasonality
412	Autocorrelation
413	Forecasting methods
DS6 Ch.6 Machine learning basics	*Needs: DS5 Ch.5 Time series & forecasting*
414	Classification
415	Clustering
416	k-means
417	Decision trees
418	Model selection
DS7 Ch.7 Deep learning & AI basics (tools)	*Needs: DS6 Ch.6 Machine learning basics*
419	Neural-network libraries
420	NLP overview (math done in S18)
DS8 Ch.8 Ethics in the data science cycle	*Needs: DS7 Ch.7 Deep learning & AI basics (tools)*
421	Privacy

422	Bias
423	Fairness
424	Responsible AI
DS9 Ch.9 Visualizing data	*Needs: DS8 Ch.8 Ethics in the data science cycle*
425	Choosing charts
426	Matplotlib
427	Seaborn
DS10 Ch.10 Reporting results	*Needs: DS9 Ch.9 Visualizing data*
428	Communicating findings
429	Reproducible reports

STAGE 18 · AI CAPSTONE

Capstone: Mathematics of Machine Learning

Topics 430–470 · 10 units

History: Rosenblatt's perceptron; backpropagation; attention and transformers. **Where it is used:** Building, training and evaluating real AI models.

X19.1	**Optimization**	*Needs: DS10 Ch.10 Reporting results · K13.4 Gradient & chain rule*
430	Convexity	
431	Gradient descent	
432	SGD	
433	Momentum	
434	Adam	
X19.2	**Regression models from scratch**	*Needs: X19.1 Optimization · R17.9 Regression*
435	Linear and logistic regression	
436	Loss functions	
437	L1/L2 regularization	
438	Bias-variance	
X19.3	**Information theory**	*Needs: X19.2 Regression models from scratch · R17.4 Discrete random variables*
439	Entropy	
440	Cross-entropy	
441	KL divergence	
X19.4	**Classical ML mathematics**	*Needs: X19.3 Information theory · L14.8 Symmetric matrices & SVD*
442	SVMs and kernels	
443	k-NN	
444	Naive Bayes	
445	PCA via SVD	
X19.5	**Neural networks & backpropagation**	*Needs: X19.4 Classical ML mathematics · L14.2 Matrix algebra*
446	Perceptron	
447	MLP	
448	Backprop as the chain rule	
449	Automatic differentiation	
X19.6	**CNNs, RNNs & transformers**	*Needs: X19.5 Neural networks & backpropagation*
450	Convolution	
451	Recurrence	
452	Attention	
453	Softmax	
454	Embeddings	
X19.7	**Probabilistic & generative models**	*Needs: X19.6 CNNs, RNNs & transformers · R17.7 Estimation*

455	MLE and MAP
456	EM
457	VAEs
458	Diffusion intuition
X19.8 Training language models	*Needs: X19.7 Probabilistic & generative models*
459	Tokenization
460	Pretraining
461	Fine-tuning
462	RLHF basics
X19.9 Evaluation & evals	*Needs: X19.8 Training language models*
463	Train/validation/test
464	Cross-validation
465	Precision, recall, F1, ROC-AUC
466	Calibration
467	LLM evals and benchmarks
468	Contamination
X19.10 Capstone project	*Needs: X19.9 Evaluation & evals*
469	Build, train and evaluate a model from scratch
470	Written report and defense

STAGE 19 · BEYOND · OPTIONAL

Beyond: No Limits

Topics 471–491 · 5 units

History: Cauchy, Weierstrass, Galois, Riemann. **Where it is used:** Research mathematics, ML theory, scientific computing.

Z20.1 Real Analysis	*Needs: X19.10 Capstone project · D15.2 Proof techniques · K12.6 Power & Taylor series*
471	Completeness
472	Sequences
473	Continuity
474	Differentiability
475	Riemann integral
476	Uniform convergence
Z20.2 Abstract Algebra	*Needs: Z20.1 Real Analysis · D15.5 Number theory & cryptography*
477	Groups
478	Rings
479	Fields
480	Galois theory introduction
Z20.3 Numerical Analysis	*Needs: Z20.2 Abstract Algebra · L14.8 Symmetric matrices & SVD*
481	Floating point
482	Root finding
483	Interpolation
484	Numerical linear algebra
Z20.4 Convex Optimization	*Needs: Z20.3 Numerical Analysis · X19.1 Optimization*
485	Convex sets
486	Duality
487	KKT conditions
488	Interior-point methods
Z20.5 Complex Analysis	*Needs: Z20.4 Convex Optimization · Z20.1 Real Analysis*
489	Analytic functions
490	Cauchy's theorem
491	Residues

The journey has no upper limit.

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